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Even and odd q-coherent states and their optical statistics properties

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Abstract. We construct explicitly even and odd q-coherent states (qCSs), which are proved to form a representation of the quantum Heisenberg-Weyl algebra, and study their properties. It is shown that optical statistics properties of the even and odd qCSs are very different. We find that an even qCS exhibits squeezing but no antibunching effect and an odd qCS has antibunching effect but no squeezing for all finite-q values.

1. Introduction

In recent years, much work has been devoted to quantum group versions of usual Lie (super) algebras, i.e. quantum groups [1-3], and their applications to integrable systems, inverse scattering problems and conformal field theory (see [4] and references therein). More recently coherent states of quantum algebras (qCSs) have attracted a lot of attention due to their possible applications in various branches of physics and mathematical physics [6]. A qCS of quantum Heisenberg-Weyl algebra (qHWA) [1], which is an eigenstate of the q-boson annihilation operator, has been studied in great detail by many authors [1, 7-9] and its applications to some concrete physical problems [10-12] have been explored. General CSs for quantum algebra $SU_q(2)$ [13] have also constructed, and extended to the quantum SU(2) superalgebra [14].

The conventional even and odd CSs [15] are two orthogonal eigenstates of the square of the boson annihilation operator. They form a complete Hilbert space, which is a representation of the HWA. It has also been shown that they are associated with non-classical properties of quantum light fields [16-18], and may play an important role in quantum optics [18, 19]. Therefore it is useful to study the even and odd q-CSs. On the other hand, such an investigation may give some new insight into the problem of the physical meaning of the deformation parameter q, which is, until now, still unclear [20].

The purpose of the present paper is to construct the even and odd qCSs and study their properties. Then we investigate their two important optical statistics properties-squeezing and antibunching-in which we have in mind that the q-boson annihilation and creation operators represent a single mode of the q-electromagnetic field.

2. Even and odd coherent states

As is well known, the conventional boson annihilation operator a, creation operator $a^\dagger$ and identity operator I satisfy the commutation relations of the HWA: $[a, a^\dagger]=I$. The corresponding number operator is defined by $N=a^\dagger a$, and has normalized eigenvectors $|n\rangle$ for the eigenvalues $n=0, 1, 2, \ldots$.

The conventional even and odd CSs [15], denoted by $|z\rangle_e$ and $|z\rangle_o$ respectively, may be defined in the form,

$$|z\rangle_e = N_e(z)\cosh(za^\dagger)|0\rangle = N_e(z)\sum_{n=0}^{\infty}\frac{z^{2n}}{\sqrt{(2n)!}}|2n\rangle \tag{1a}$$

$$|z\rangle_o = N_o(z)\sinh(za^\dagger)|0\rangle = N_o(z)\sum_{n=0}^{\infty}\frac{z^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle \tag{1b}$$

where z is a complex number and the normalization constants are given by

$$N_e(z) = (\cosh(z\bar{z}))^{-1/2} \tag{2a}$$

$$N_o(z) = (\sinh(z\bar{z}))^{-1/2}. \tag{2b}$$

From the definition of the even and odd CSs, it can be shown that they are eigenstates of the square of the annihilation operator, i.e.

$$a^2|z\rangle_e = z^2|z\rangle_e \tag{3a}$$

$$a^2|z\rangle_o = z^2|z\rangle_o. \tag{3b}$$

It is obvious that the even CS and the odd CS are orthogonal to each other

$${}_e\langle z'|z\rangle_o = 0. \tag{4}$$

However, the even CSs and the odd CSs are non-orthogonal. They satisfy the orthogonality relations,

$${}_e\langle z'|z\rangle_e = N_e(z')N_e(z)\cosh(z\bar{z}') \tag{5a}$$

$${}_o\langle z'|z\rangle_o = N_o(z')N_o(z)\sinh(z\bar{z}'). \tag{5b}$$

The even CSs and the odd CSs can be transformed into each other by the action of the annihilation operator a, namely,

$$a|z\rangle_e = z\tanh^{-1/2}(z\bar{z})|z\rangle_o \tag{6a}$$

$$a|z\rangle_o = z\coth^{-1/2}(z\bar{z})|z\rangle_e. \tag{6b}$$

This means that the annihilation operator a acts as a rotation operator between $|z\rangle_e$ and $|z\rangle_o$.

Although the even (odd) CSs cannot form a complete set, the even CSs together with the odd CSs constitute a complete Hilbert space, and satisfy the following complete relation,

$$\frac{1}{\pi}\int d^2z\, e^{(-z\bar{z})}\{\cosh(z\bar{z})|z\rangle_e\,{}_e\langle z| + \sinh(z\bar{z})|z\rangle_o\,{}_o\langle z|\} = I \tag{7}$$

where the integral is taken over the entire complex plane, with $d^2z = d(\mathrm{Re}\, z)\, d(\mathrm{Im}\, z)$.

3. Even and odd q-coherent states

The qHWA [1] is generated by q-creation operator $a_q^\dagger$, q-annihilation operator a_q and a q-number operator N_q. These operators satisfy the commutation relations

$$a_q a_q^\dagger - q a_q^\dagger a_q = q^{-N_q} \tag{8}$$

$$[N_q, a_q] = -a_q \qquad [N_q, a_q^\dagger] = a_q^\dagger. \tag{9}$$

In what follows we shall concentrate on $0<q<1$; the range $1<q<\infty$ then corresponds to the replacement $q \leftrightarrow q^{-1}$ throughout. The operators a_q, $a_q^\dagger$ and N_q act in a Hilbert space with the basis $|n\rangle_q$ ($n=0, 1, 2, \ldots$), such that

$$a_q|0\rangle_q = 0 \tag{10}$$

$$|n\rangle_q = \frac{(a_q^\dagger)^n}{([n]_q!)^{1/2}}|0\rangle_q \tag{11}$$

where the q-factorial $[n]_q! = [n]_q[n-1]_q \ldots [1]_q$ with the q-number

$$[x]_q = \frac{q^x - q^{-x}}{q - q^{-1}}. \tag{12}$$

Their actions on the basis vectors are given by

$$a_q^\dagger|n\rangle_q = \sqrt{[n+1]_q}\,|n+1\rangle_q \tag{13}$$

$$a_q|n\rangle_q = \sqrt{[n]_q}\,|n-1\rangle_q. \tag{14}$$

The resolution of unity in the Hilbert space is written as

$$I = \sum_{n=0}^{\infty} |n\rangle_{q\,q}\langle n|. \tag{15}$$

We now define an even and odd qCS as

$$|z\rangle_q^e = N_q^e(z) \cosh_q(z a_q^\dagger)|0\rangle_q \tag{16a}$$

$$|z\rangle_q^o = N_q^o(z) \sinh_q(z a_q^\dagger)|0\rangle_q. \tag{16b}$$

Where $N_q^e(z)$ and $N_q^o(z)$ are normalization constants to be determined, and we have introduced two q-functions,

$$\cosh_q x = \tfrac{1}{2}(e_q^x + e_q^{-x}) = \sum_{n=0}^{\infty} \frac{x^{2n}}{[2n]_q!} \tag{17a}$$

$$\sinh_q x = \tfrac{1}{2}(e_q^x - e_q^{-x}) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{[2n+1]_q!} \tag{17b}$$

where we have used the q-exponential function

$$e_q^x = \sum_{n=0}^{\infty} \frac{x^n}{[n]_q!}. \tag{18}$$

Substituting (17) into (16) and making use of (13), one can rewrite the even and odd qCSs as

$$|z\rangle_q^e = N_q^e(z) \sum_{n=0}^{\infty} \frac{z^{2n}}{\sqrt{[2n]_q!}}|2n\rangle_q \tag{19a}$$

$$|z\rangle_q^o = N_q^o(z) \sum_{n=0}^{\infty} \frac{z^{2n+1}}{\sqrt{[2n+1]_q!}}|2n+1\rangle_q. \tag{19b}$$

We require that the even and odd qCSs are normalized in the form,

$$ {}^{e}_{q}\langle z|z\rangle^{e}_{q}=1 \tag{20a}$$

$$ {}^{o}_{q}\langle z|z\rangle^{o}_{q}=1. \tag{20b}$$

Then, the normalization constants are given by

$$N^{e}_{q}(z)=(\cosh_q(z\bar{z}))^{-1/2} \tag{21a}$$

$$N^{o}_{q}(z)=(\sinh_q(z\bar{z}))^{-1/2}. \tag{21b}$$

From equation (19) it follows that

$$ {}^{e}_{q}\langle z'|z\rangle^{e}_{q}=N^{e}_{q}(z')N^{e}_{q}(z)\cosh_q(z\bar{z}') \tag{22a}$$

$$ {}^{o}_{q}\langle z'|z\rangle^{o}_{q}=N^{o}_{q}(z')N^{o}_{q}(z)\sinh_q(z\bar{z}') \tag{22b}$$

$$ {}^{e}_{q}\langle z'|z\rangle^{o}_{q}=0. \tag{22c}$$

This means that the even (odd) qCSs are non-orthogonal; however, the even qCSs and the odd qCSs are orthogonal to each other.

As is well known, the core of such a system for CSs is their completeness. In the present case, it can be shown that the even (odd) qCSs do not form a complete set. However, the even qCSs together with the odd qCSs build a complete Hilbert space. Furthermore their complete relation holds in the following form,

$$\frac{[2]}{2\pi}\int d^2_q z\, e_q^{-z\bar{z}}\{\cosh_q(z\bar{z})|z\rangle^{e}_{q}\,{}^{e}_{q}\langle z|+\sinh_q(z\bar{z})|z\rangle^{o}_{q}\,{}^{o}_{q}\langle z|\}=I \tag{23}$$

where $d^2_q z=r\,d_q r\,d\theta$ with $z=r\,e^{i\theta}$, so the integral over the variable r is a q-integration [8, 9] while the integral over $d\theta$ is a normal integration.

Proof. Substituting (19) and (21) into (23), the left-hand side of (23) may be written as

$$\begin{aligned}
&\frac{[2]}{2\pi}\int d^2_q z\, e_q^{-z\bar{z}}\sum_{n,m}\left\{\frac{z^{2n}\bar{z}^{2m}}{\sqrt{[2n]_q![2m]_q!}}|2n\rangle_{q\,q}\langle 2m|\right.\\
&\qquad\left.+\frac{z^{2n+1}\bar{z}^{2m+1}}{\sqrt{[2n+1]_q![2m+1]_q!}}|2n+1\rangle_{q\,q}\langle 2m+1|\right\}\\
&=\frac{[2]}{2\pi}\int d^2_q z\, e_q^{-z\bar{z}}\sum_{n,m}\frac{z^{n}\bar{z}^{m}}{\sqrt{[n]_q![m]_q!}}|n\rangle_{q\,q}\langle n|\\
&=\frac{[2]}{2\pi}\int_0^{\zeta} r\,d_q r\int_0^{2\pi} d\theta\, e_q^{-r^2}\sum_{n,m}\frac{r^{n+m}e^{i(n-m)\theta}}{\sqrt{[n]_q![m]_q!}}|n\rangle_{q\,q}\langle m|\\
&=[2]\sum_{n=0}^{\infty}\int_0^{\zeta} d_q r\, e_q^{-r^2}\, r^{2n+1}\frac{1}{[n]_q!}|n\rangle_{q\,q}\langle n|=I
\end{aligned}$$

where we have used the q-Euler's formula for $\Gamma(x)$ function [8]

$$\int_0^{\zeta} d_q x\, e_q^{-x}x^n=[n]_q! \tag{24}$$

where ζ is the largest zero of the q-exponential function e_q^{-x}. Note that this q-Euler formula is different from the one found in [23].

With the help of (14) and (19), one can obtain

$$a_q^2|z\rangle_q^e = z^2|z\rangle_q^e \tag{25a}$$

$$a_q^2|z\rangle_q^o = z^2|z\rangle_q^o. \tag{25b}$$

This means that $|z\rangle_q^e$ and $|z\rangle_q^o$ are two orthogonal eigenstates of the square of the q-annihilation operator.

From (14) and (19), it is straightforwatd to obtain

$$a_q|z\rangle_q^e = z(\tanh_q(z\bar{z}))^{1/2}|z\rangle_q^o \tag{26a}$$

$$a_q|z\rangle_q^o = z(\coth_q(z\bar{z}))^{1/2}|z\rangle_q^e \tag{26b}$$

where

$$\tanh_q x = \frac{e_q^x - e_q^{-x}}{e_q^x + e_q^{-x}} \tag{27a}$$

$$\coth_q x = \frac{e_q^x + e_q^{-x}}{e_q^x - e_q^{-x}} \tag{27b}$$

so that the even and odd qCSs can be transformed by the action of the q-annihilation operator a_q.

As a consequence of equations (23) and (26), the even and odd qCSs together give rise to a representation of the qHWA.

Finally, let us observe the relation between the even and odd qCSs, and the Glauber-type qCS [6–9] defined by

$$|z\rangle_q = N_q(z) \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{[n]_q!}} |n\rangle_q \tag{28}$$

where the normalization constant is given by

$$N_q(z) = (e_q^{z\bar{z}})^{-1/2}. \tag{29}$$

From equations (19) and (27) it follows that

$$|z\rangle_q^e = \tfrac{1}{2} N_q^{-1}(z) N_q^e(z)(|z\rangle_q + |-z\rangle_q) \tag{30a}$$

$$|z\rangle_q^o = \tfrac{1}{2} N_q^{-1}(z) N_q^o(z)(|z\rangle_q - |-z\rangle_q) \tag{30b}$$

which indicate that the even and odd qCSs can be expanded nonlinearly in terms of the Glauber-type qCSs. Apparently, the even and odd qCSs, and the Glauber-type qCSs are non-trivially different.

As expected, the even and odd qCSs become the conventional even and odd CSs in the limit $q \to 1$.

4. Optical statistics properties of the even and odd qCSs

In this section, we shall investigate some optical statistics properties of the even and odd qCSs concerning quantum mechanical effects of light, squeezing and antibunching properties.

In analogy with the definition of squeezing for the conventional single mode of the electromagnetic field [21] we introduce the q-squeezing for the q-electromagnetic field, assumed to be expressed in terms of the q-annihilation and creation operators a_q, $a_q^\dagger$ in the standard way

$$X_1^q = \tfrac{1}{2}(a_q^\dagger + a_q) \tag{31a}$$

$$X_2^q = \tfrac{1}{2}i(a_q^\dagger - a_q). \tag{31b}$$

These operators obey the commutation relation

$$[X_1^q, X_2^q] = \tfrac{1}{2}i[a_q, a_q^\dagger] \tag{32}$$

and as a result, satisfy the uncertainty relation

$$\langle(\Delta X_1^q)^2\rangle\langle(\Delta X_2^q)^2\rangle \geqslant \tfrac{1}{4}|\langle[X_1^q, X_2^q]\rangle|^2 \tag{33}$$

where the variance $\langle(\Delta X_i^q)^2\rangle$ is defined as $\langle(\Delta X_i^q)^2\rangle = \langle(X_i^q)^2\rangle - (\langle X_i^q\rangle)^2$ $(i=1,2)$.

We will call a state q-squeezing in the X_1^q variable if

$$_q\langle(\Delta X_1^q)^2\rangle_q < \tfrac{1}{2}|_q\langle[X_1^q, X_2^q]\rangle_q| \tag{34}$$

and similarly for X_2^q.

We now calculate the various expectation values appearing in the above equations. From equations (13), (14) and (19), one may get

$$\begin{aligned} {}_q^e\langle z|a_q^\dagger a_q|z\rangle_q^e &= z\bar{z}\tanh_q(z\bar{z}) \\ {}_q^e\langle z|a_q a_q^\dagger|z\rangle_q^e &= qz\bar{z}\tanh_q(z\bar{z}) + \frac{\cosh_q(q^{-1}z\bar{z})}{\cosh_q(z\bar{z})} \end{aligned} \tag{35}$$

$$\begin{aligned} {}_q^o\langle z|a_q^\dagger a_q|z\rangle_q^o &= z\bar{z}\coth_q(z\bar{z}) \\ {}_q^o\langle z|a_q a_q^\dagger|z\rangle_q^o &= qz\bar{z}\coth_q(z\bar{z}) + \frac{\sinh_q(q^{-1}z\bar{z})}{\sinh_q(z\bar{z})} \end{aligned} \tag{36}$$

$$\begin{aligned} {}_q^e\langle z|(a_q^2 + a_q^{\dagger 2})|z\rangle_q^e &= {}_q^o\langle z|(a_q^2 + a_q^{\dagger 2})|z\rangle_q^o = z^2 + \bar{z}^2 \\ {}_q^e\langle z|a_q^\dagger|z\rangle_q^e &= {}_q^o\langle z|a_q^\dagger|z\rangle_q^o = 0. \end{aligned} \tag{37}$$

From the above it follows that

$${}_q^e\langle z|X_1^q|z\rangle_q^e = 0 \tag{38}$$

$${}_q^e\langle z|(X_1^q)^2|z\rangle_q^e = \tfrac{1}{2}r^2\{\cos 2\theta + \tfrac{1}{2}(1+q)\tanh_q r^2\} + \frac{1}{4}\frac{\cosh_q(q^{-1}r^2)}{\cosh_q r^2} \tag{39}$$

$${}_q^o\langle z|X_1^q|z\rangle_q^o = 0 \tag{40}$$

$${}_q^o\langle z|(X_1^q)^2|z\rangle_q^o = \tfrac{1}{2}r^2\{\cos 2\theta + \tfrac{1}{2}(1+q)\coth_q r^2\} + \frac{1}{4}\frac{\sinh_q(q^{-1}r^2)}{\sinh_q r^2} \tag{41}$$

so that

$${}_q^e\langle z|(\Delta X_1^q)^2|z\rangle_q^e = \tfrac{1}{2}r^2\{\cos 2\theta + \tfrac{1}{2}(1+q)\tanh_q r^2\} + \frac{1}{4}\frac{\cosh_q(q^{-1}r^2)}{\cosh_q r^2} \tag{42}$$

$${}_q^o\langle z|(\Delta X_1^q)^2|z\rangle_q^o = \tfrac{1}{2}r^2\{\cos 2\theta + \tfrac{1}{2}(1+q)\coth_q r^2\} + \frac{1}{4}\frac{\sinh_q(q^{-1}r^2)}{\sinh_q r^2} \tag{43}$$

where we have taken $z = r\,e^{i\theta}$.

Similarly, one can get the variances in X_2^q,

$$ {}_q^e\langle z|(\Delta X_2^q)^2|z\rangle_q^e = -\tfrac{1}{2}r^2\{\cos 2\theta - \tfrac{1}{2}(1+q)\tanh_q r^2\} + \frac{1}{4}\frac{\cosh_q(q^{-1}r^2)}{\cosh_q r^2} \tag{44}$$

$$ {}_q^o\langle z|(\Delta X_2^q)^2|z\rangle_q^o = \tfrac{1}{2}r^2\{\cos 2\theta + \tfrac{1}{2}(1+q)\coth_q r^2\} + \frac{1}{4}\frac{\sinh_q(q^{-1}r^2)}{\sinh_q r^2} \tag{45}$$

and

$$ {}_q^e\langle z|[X_1^q, X_2^q]|z\rangle_q^e = \tfrac{1}{2}i\left\{(q-1)r^2\tanh_q r^2 + \frac{\cosh_q(q^{-1}r^2)}{\cosh_q r^2}\right\} \tag{46}$$

$$ {}_q^o\langle z|[X_1^q, X_2^q]|z\rangle_q^o = \tfrac{1}{2}i\left\{(q-1)r^2\coth_q r^2 + \frac{\sinh_q(q^{-1}r^2)}{\sinh_q r^2}\right\}. \tag{47}$$

To analyse q-squeezing properties, we consider

$$ {}_q^e\langle z|(\Delta X_1^q)^2|z\rangle_q^e - \tfrac{1}{2}|{}_q^e\langle z|[X_1^q, X_2^q]|z\rangle_q^e| = \tfrac{1}{2}r^2\{\cos 2\theta + \tanh_q r^2\} \tag{48}$$

$$ {}_q^o\langle z|(\Delta X_1^q)^2|z\rangle_q^o - \tfrac{1}{2}|{}_q^o\langle z|[X_1^q, X_2^q]|z\rangle_q^o| = \tfrac{1}{2}r^2\{\cos 2\theta + \coth_q r^2\}. \tag{49}$$

Because of $\tanh_q r^2 < 1$, equation (48) shows that the following inequality holds

$$ {}_q^e\langle z|(\Delta X_1^q)^2|z\rangle_q^e < \tfrac{1}{2}|{}_q^e\langle z|[X_1^q, X_2^q]|z\rangle_q^e| \tag{50}$$

when 2θ is in the range $(\tfrac{1}{2}\pi, \pi)$.

From the above it is seen that with respect to the even qCSs there is squeezing for all finite-q values, in particular, including the $q=1$ limit case. This is very different from the case of the Glauber-type qCSs [10] in which no squeezing occurs in the $q=1$ limit.

Equation (49) leads to

$$ {}_q^o\langle z|(\Delta X_1^q)^2|z\rangle_q^o > \tfrac{1}{2}|{}_q^o\langle z|[X_1^q, X_2^q]|z\rangle_q^o| \tag{51}$$

where we have used $\coth_q r^2 > 1$. This indicates that with respect to the odd qCSs no squeezing occurs for all finite-q values.

Finally, we investigate the antibunching effect of the even and odd qCSs. As is well known, if the normalized second-order correlation function of a light field [22] $g^{(2)}(0) < 1$, one says the light field exhibits antibunching effect. In a similar way, we introduce the second-order q-correlation function for the q-light field

$$ g_q^{(2)}(0) = \frac{{}_q\langle z|(a_q^\dagger)^2(a_q)^2|z\rangle_q}{|{}_q\langle z|a_q^\dagger a_q|z\rangle_q|^2}. \tag{52}$$

It is straightforward to evaluate the second-order q-correlation function for the even and odd qCSs, respectively,

$$ g_{qe}^{(2)}(0) = \coth_q r^2 \tag{53}$$

$$ g_{qo}^{(2)}(0) = \tanh_q r^2. \tag{54}$$

Because of $\tanh_q r^2 < 1$ and $\coth_q r^2 > 1$ for all finite-q values, $g_{qe}^{(2)}(0) > 1$ and $g_{qo}^{(2)}(0) < 1$. This means that the odd qCSs exhibit antibunching effect but the even qCSs do not for all q values.

5. Concluding remarks

We have constructed the even and odd qCSs and discussed some of their properties. Although the even (odd) qCSs cannot consist of a complete set, the even qCSs together with the odd qCSs form a complete Hilbert space. From a mathematical point of view, this Hilbert space is a representation space of the qHWA. The even and odd qCSs therefore give rise to a new representation of the qHWA. We have also investigated optical statistics properties of the even and odd qCSs, and found that with the even qCSs squeezing may occur but there is no antibunching effect, however, for the odd qCSs there is antibunching but no squeezing for all finite-q values. It has been shown that the deformation parameter q may be a parameter relevant to the degree of squeezing and antibunching.

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